Instructions

Please read the previously uploaded instructions.

Exercise 1 (9 points)

A curtain is a piece of fabric that covers a window. We assume that curtains look like opposed semicircles (pleats), as in Figure 1.

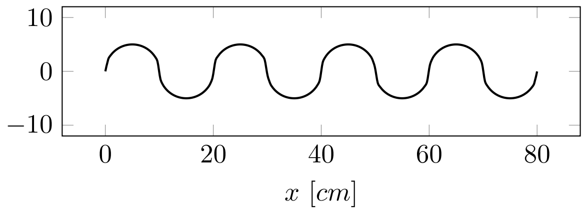

Figure 1: Undulation of a curtain. Top view.

Therefore, if one stretch the curtain, such that the pledges are flattened, the fabric is wider than just the window.

1. **(2.5pt)** For aesthetic reasons, the radius of the pleats is adjusted to match the width of the window with an exact number of pleats, as can be seen in Figure 1. The radius should be as close to $5cm$ as possible. Produce a function length(W) which takes the width of the window (W) and returns the total (i.e., stretched) length of the curtain.

The curtain is made by attaching vertical strips of fabric one next to another until reaching the desired length. The fabric is sold by entire strips. Strips width is fixed at $150cm$. If a narrower strip is needed, one buys an entire one and clips the leftover. Figure 2 illustrates this.

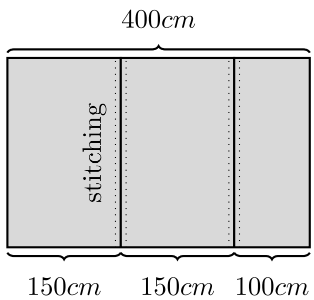

Figure 2: Curtain made of vertical strips. Front view of the unfolded fabric.

2. **(2.5pt)** Assuming that the function length(W) work, produce a function surface(W, H) which takes the width (W) and height (H) of the window and returns the surface that needs to be bough to **manufacture the curtain**.

Hooks are used to hang the curtain. A hook is placed at the beginning of every pledge, and also at the end of the last one. Figure 3 gives an example for a curtain with 5 pledges.

3. **(2.0pt)**Assuming all the above functions work, produce a function **hooks(W)** that takes the width of the window (W) and prints a list of the positions along the curtain where a hook needs to be sewed (including both ends).

Now, instead of semicircles, one can have different pleat shapes by defining $y = f(x)$, as far as $f(x)$ is periodic. Figure 4 shows an example. To compute the length of the curve given W, the width of the x-axis, one needs to perform the following integral:

$$L = \int_0^W \sqrt{1 + f'(x)^2} dx \tag{1}$$

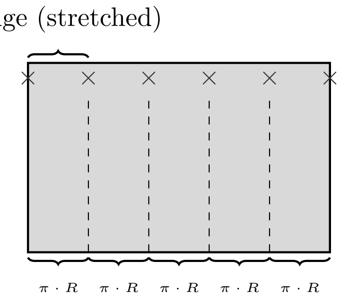

Figure 3: Hooks sewed to the curtain ($\times$). Front view of the unfolded fabric.

where $f'(x)$ is the derivative of $f(x)$. This integral has not necessearily a closed form solution.

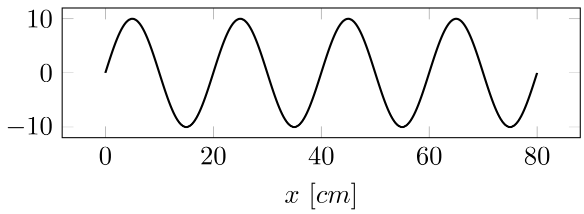

Figure 4: Undulation defined as $y = 10 \sin(2\pi \cdot x/20)$. Top view.

4. **2.0pt** Produce a program `curve(W, g)` which takes `W`, the width along the x-axis and the function `g` $= f'(x)$ and returns the numerical integration by mean of the Riemann sum of Equation (1).